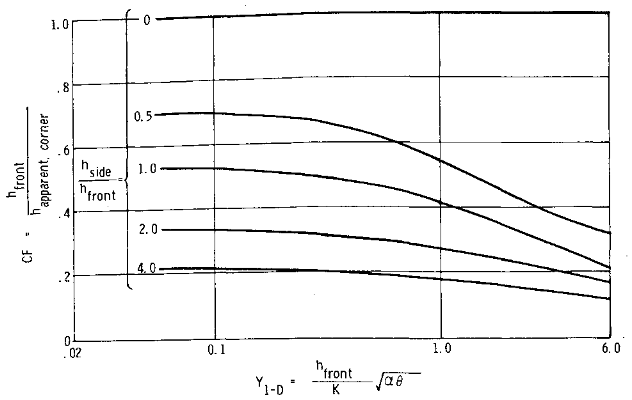

Fig. 3 Correction factor at corner-angle = 90°.

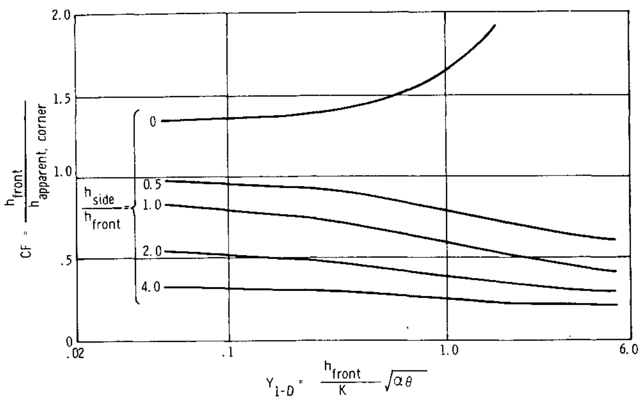

Fig. 4 Correction factor at corner-angle = 120°.

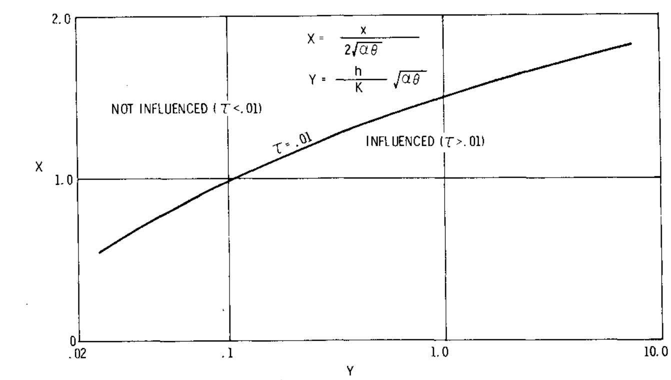

Fig. 5 Region of influence-one-dimensional solution (assumed to apply to 2-D solutions if x is taken as distance from corner.

lower apparent heat transfer coefficient than the one-dimensional solution.

The correction factor to apply to heat transfer coefficients obtained from one-dimensional solutions is given by the following relationship:

$$CF = \frac{h_{front}}{h_{apparent, corner}} = \left[\frac{Y_{2-D}}{Y_{1-D}}\right]_{\tau = constant} \tag{3}$$

Computed correction factors are presented in Figs. 3 and 4 as a function of Y_{1-D}.

It is apparent from the results that sizable errors may be realized by treating corners as one-dimensional problems. The error, of course, diminishes with distance from the corner. An estimate of the distance from the corner for which two-dimensional effects become negligible is obtained by solving Eq. (1) for $\tau = 0.01$. The results of this computation are shown in Fig. 5. An examination of actual results showed the rate of temperature propagation from the corner to be less than that given above.

This study assumed the recovery temperatures for the front and side surfaces were equal. In general, this is not the case. Higher recovery temperatures on the side surface than on the front will result in larger correction factors than those shown in Figs. 3 and 4. Conversely, lower side surface recovery temperatures will result in smaller correction factors.

References

[1] Jones, R. A. and Hunt, J. L., "Use of Fusible Temperature Indicators for Obtaining Quantitative Aerodynamic Heat-Transfer Data," TR R-230, Feb. 1966, NASA.

[2] Hunt, J. L., "Heat Transfer to Four Fineness-Ratio-1.6 Hexagonal Prisms with Various Corner Radi at Mach 6," TM X-2446, March 1972, NASA.

[3] Hunt, J. L., Pitts, J. L. and Richie, C. B., "Application of Phase-Change Technique to Thin Sections with Heating on Both Surfaces," TN D-7193, Aug. 1973, NASA.

[4] Creel, T. R., "Experimental Investigation at Mach 8 of the Effects of Projections and Cavities on Heat Transfer to a Model of the Viking Aeroshell," TM X-2941, April 1974, NASA.

[5] Carslae, H. S. and Jaeger, J. C., *Conduction of Heat in Solids*, 2nd ed., Oxford at the Clarendon Press, 1959, pp. 70-72, pp. 475-476.

Low Reynolds Number Effect on Hypersonic Lifting Body Turbulent Boundary Layers

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Nomenclature

A_*	= van Driest damping constant, 26.0
D	= van Driest exponential damping function for the near wall region
k	= inner law mixing-length constant, 0.435
$\mathcal{L}$	= low Reynolds number mixing-length parameter
L	= reference length
$\mathcal{L}$	= mixing length
M_∞	= freestream Mach number
Pr_t	= turbulent Prandtl number, 0.90
$Re_{e,\theta}$	= local Reynolds number based on inviscid edge conditions and boundary-layer momentum thickness
Re_∞/ft	= freestream unit Reynolds number
St_∞	= local Stanton number based on freestream conditions

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Index categories: Boundary Layers and Convective Heat Transfer—Turbulent; Supersonic and Hypersonic Flow.

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$St_{*,ref}$ = reference Stanton number
T_w = wall temperature
$V_{\tau,w}$ = local friction velocity based on wall conditions
x = crossflow coordinate normal to body centerline
x^* = crossflow coordinate distance from body centerline to location of sonic point
y = coordinate normal to body surface
$y_{\mathcal{L}}$ = characteristic thickness of boundary layer
z = streamwise coordinate along body centerline
α = angle of attack
δ_w^+ = local boundary-layer thickness expressed in law-of-the-wall coordinates based on wall conditions
θ = boundary-layer momentum thickness
λ = outer law mixing-length constant, 0.09
μ_w, ρ_w, τ_w = wall viscosity, density, and shearing stress, respectively

Introduction

RECENT analytical studies by Adams and Martindale[1,2] have revealed the following characteristics of compressible turbulent boundary-layer flow along the windward centerline of lifting body configurations at high incidence angles under perfect gas hypersonic wind tunnel conditions: 1) low edge Mach number (order of one to two); 2) low edge Reynolds number based on boundary-layer momentum thickness (order of 500 to 2000); 3) wall temperature on the order of the boundary-layer edge temperature; and 4) crossflow-dominated boundary-layer flow in the sense that "strip theory" as defined in Refs. 1 and 2 is applicable.

It is now fairly well known[3-10] that in the case of two-dimensional turbulent boundary-layer flow under either incompressible or compressible conditions, large increases in eddy viscosity can occur under so-called low Reynolds number conditions (where the Reynolds number is defined in terms of edge conditions and boundary-layer momentum thickness, i.e., $Re_{e,\theta}$). As shown in Fig. 11 of Ref. 3, the traditional momentum-defect law fails for values of $Re_{e,\theta}$ less than about 6000, with the wake component disappearing entirely—and rather abruptly—by the time $Re_{e,\theta}$ has reached a value in the neighborhood of 500. Since the $Re_{e,\theta}$ values for turbulent boundary-layer flow on the windward centerline of lifting body configurations at high incidence angles under hypersonic perfect gas conditions are within this range (see previous paragraph), the present Note will report modifications to the eddy viscosity model of Adams[1,2] to more properly account for low Reynolds number effects in turbulence modeling.

Analysis

The two-layer (inner-outer) mixing-length model used in the three-dimensional scalar eddy viscosity formulation of Adams[1,2] is basically that of Patankar and Spalding,[11] namely

$$\mathcal{L} = kyD, \text{ for } 0<y\leq \lambda y_{\mathcal{L}}/k \tag{1}$$

and

$$\mathcal{L} = \lambda y_{\mathcal{L}}, \text{ for } \lambda y_{\mathcal{L}} = k<y \tag{2}$$

where the values for the numerical constants are taken to be $k=0.435$ and $\lambda=0.09$. The value of y at the point where the velocity in the boundary layer is equal to 0.99 of the velocity at the boundary-layer outer edge is used to define the distance $y_{\mathcal{L}}$. The so-called van Driest[12] exponential damping, D, is applied for the near-wall region in conjunction with the suggestion by Patankar and Spalding[11] that the local value of the total shear stress be used instead of the wall value as originally recommended by van Driest. The van Driest damping constant, A_*, is taken to be 26.0, following the original van Driest proposal.[12] Turbulence influence on boundary-layer heat flux is modeled through a turbulent Prandtl number, Pr_t, which is taken to remain constant at the value 0.90 across the entire boundary layer, as recommended by Patankar and Spalding.[11]

For the present study, which is limited to low supersonic edge Mach number flows because of the high angle-of-attack condition discussed in the Introduction, the parameter k and the van Driest damping constant are assumed to be universal constants. Low Reynolds number effects are allowed by modifying the outer-layer mixing length given by Eq. (2) to the form

$$\mathcal{L} = \mathcal{L}\lambda y_{\mathcal{L}}, \text{ for } \mathcal{L}\lambda y_{\mathcal{L}}/k<y \tag{3}$$

where λ and $y_{\mathcal{L}}$ have been previously defined. The parameter $\mathcal{L}$ reflects the low Reynolds number phenomenon and is defined as

$$\mathcal{L} = 1.0+\exp(-2.034\ \ln\delta_w^+ + 10.9) \tag{4}$$

with δ_w^+ the local boundary-layer thickness expressed in conventional law-of-the-wall coordinates based on wall conditions

$$\delta_w^+ = (\rho_w V_{\tau,w} y_{\mathcal{L}})/\mu_w \tag{5}$$

where $V_{\tau,w}$ is the so-called wall friction velocity given by

$$V_{\tau,w} = (\tau_w/\rho_w)^{1/2} \tag{6}$$

The form of Eq. (4) is analogous to Eq. (27) in Shamroth and McDonald[13] and, in fact, reduces essentially to their equation if one uses Table 2 in Ref. 3 for converting δ_w^+ to $Re_{e,\theta}$.

The choice of δ_w^+ as the low Reynolds number correlating parameter follow Bushnell and Morris.[7] However, the

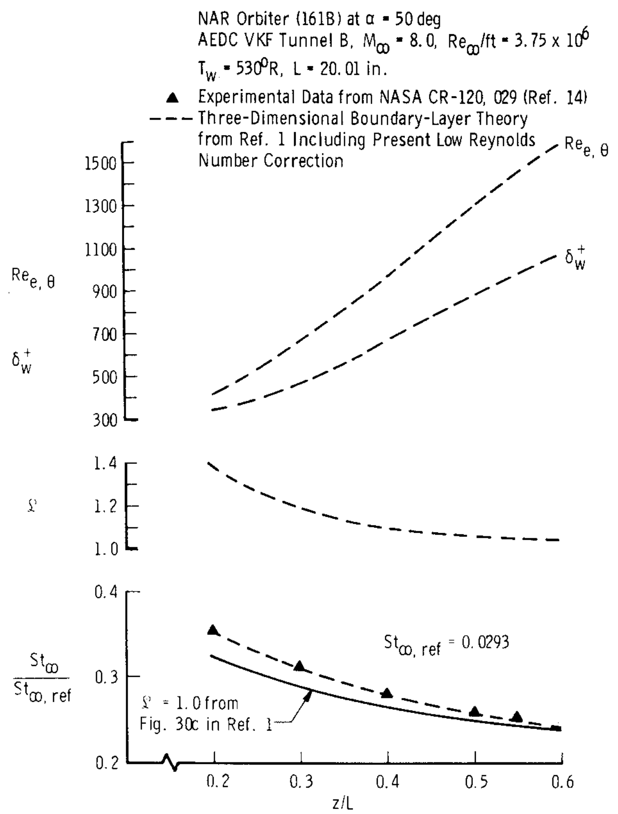

Fig. 1 **Windward centerline turbulent boundary-layer parameters.**

present use of δ_w^+ is more subtle. The three-dimensional scalar eddy viscosity model of Adams[1,2] can easily be modified to include low Reynolds number effects on three-dimensional turbulent boundary layers. This can be accomplished by application of Eqs. (1), (3), and (4) in conjunction with inclusion of the local total or resultant wall shearing stress in the definition of the wall friction velocity, Eq. (6). If one were to specify $\mathcal{L}$ in terms of $Re_{e,\theta}$ it is not clear how $Re_{e,\theta}$ would be defined to properly reflect the three-dimensional nature of the flow.

Results and Discussion

The previously discussed low Reynolds number modification to the outer-layer mixing length has been used in conjunction with the "strip theory" approach[1,2] to compute the windward suface turbulent boundary-layer flow on the North American Rockwell (NAR) Delta Wing Orbiter Configuration 161B at 50° angle of attack under AEDC-VKF Hypersonic Wind Tunnel (B) conditions for which experimentally measured surface heat-transfer rates via the thin-skin thermocouple technique are available in the open literature.[14,15] Full details concerning the application of "strip theory" to this particular configuration are given in Refs. 1 and 2.

As shown in the lower portion of Fig. 1, inclusion of the low Reynolds number effect improves the agreement between "strip theory" and experiment with respect to windward centerline heat transfer as reflected in the Stanton number. Low Reynolds number effects are more dominant in the nose region of the body. At $z/L=0.2$, $\mathcal{L}=1.38$, which is reflected in a seven-percent increase in heat-transfer rate. At $z/L=0.5$, $\mathcal{L}=1.05$, which represents only about a 2% increase in heat-transfer rate. As shown in the upper portion of Fig. 1, $Re_{e,\theta}$ varies from 400 to 1600, i.e., a low edge Reynolds number based on boundary-layer momentum thickness; the corresponding values of δ_w^+ range from 300 to 1100.

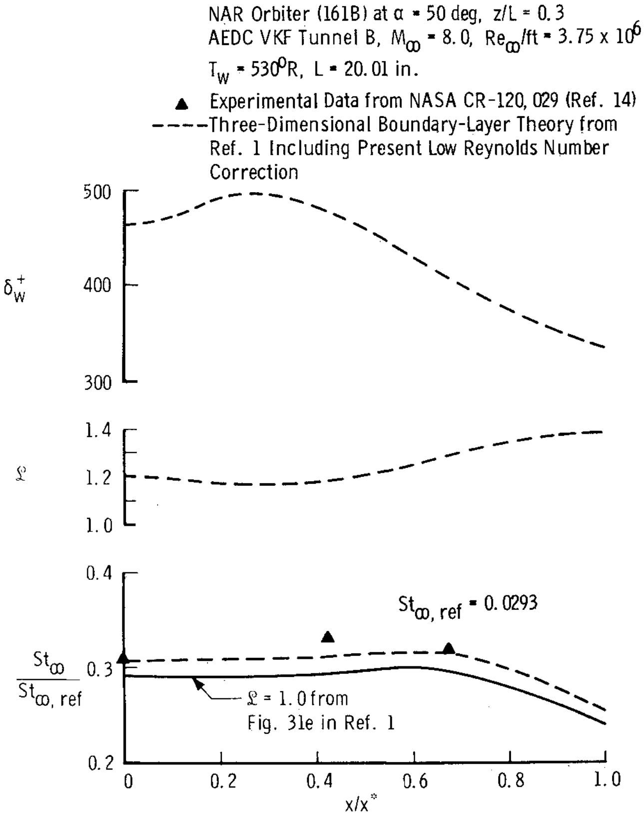

Fig. 2 Spanwise turbulent boundary-layer parameters.

Low Reynolds number effects on the three-dimensional turbulent boundary layer in terms of spanwise boundary-layer parameters at the body station location $z/L=0.3$ are given in Fig. 2. Inclusion of low Reynolds number effects improves the agreement between "strip theory" and experiment with respect to spanwise heat transfer as reflected in the Stanton number. The interesting point to note from Fig. 2 is that low Reynolds number effects as controlled by the numerical value of $\mathcal{L}$ can be of increasing importance at off-centerline locations because of the behavior of δ_w^+ in regions of highly favorable pressure gradients. From $x/x^* \approx 0.3$ outward, the three-dimensional turbulent boundary layer is being substantially accelerated (see Sec. 3.3 in Ref. 1); this acceleration results in a continuously decreasing value for δ_w^+ as shown in the upper portion of Fig. 2.

In the present Note it has been tacitly assumed that δ_w^+ is the correct correlation parameter to account for low Reynolds number effects on the outer-layer mixing length in terms of the basic Patankar-Spalding eddy viscosity model. The present success in using this parameter for high angle-of-attack lifting body windward surface applications under hypersonic wind tunnel conditions should not be interpreted as meaning that δ_w^+ is the correct correlation parameter for all low Reynolds number compressible turbulent boundary-layer flows. It may well be that $Re_{e,\theta}$ or some other parameter is more appropriate under certain conditions.

References

[1] Adams, J. C., Jr. and Martindale, W. R., "Hypersonic Lifting Body Windward Surface Flow-Field Analysis for High Angles of Incidence," AEDC-TR-73-2 (AD756499), Feb. 1973, Arnold Engineering Development Center, Arnold Air Force Station, Tenn.

[2] Adams, J. C., Jr. and Martindale, W. R., "Engineering Analysis of Hypersonic Lifting Body Windward Surface Inviscid and Viscous Flow Fields at High Angles of Incidence," AIAA Paper 73-637, Palm Springs, Calif., 1973.

[3] Coles, D., "The Turbulent Boundary Layer in a Compressible Fluid," Rept. R-403-PR, Sept. 1962, Rand Corp., Santa Monica, Calif.

[4] Herring, H. J. and Mellor, G. L., "A Method of Calculating Compressible Turbulent Boundary Layers," CR-1144, Sept. 1968, NASA.

[5] Simpson, R. L., "Characteristics of Turbulent Boundary Layers at Low Reynolds Numbers with and without Transpiration," *Journal of Fluid Mechanics*, Vol. 42, Pt. 4, July 1970, pp. 769-802.

[6] McDonald, H., "Mixing Length and Kinematic Eddy Viscosity in a Low Reynolds Number Boundary Layer," Rept. J214453-1, Sept.

[7] Bushnell, D. M. and Morris, D. J., "Shear-Stress, Eddy-Viscosity, and Mixing-Length Distributions in Hypersonic Turbulent Boundary Layers," TM X-2310, Aug. 1971, NASA.

[8] Huffman, D. G. and Bradshaw, P., "A Note on von Karman's Constant in Low Reynolds Number Turbulent Flows," *Journal of Fluid Mechanics,* Vol. 53, Pt. 1, May 1972, pp. 45-60.

[9] Cebeci, T., "Kinematic Eddy Viscosity at Low Reynolds Numbers," *AIAA Journal,* Vol. 11, Jan. 1973, pp. 102-104.

[10] Bushnell, D. M. and Alston, D. W., "Calculation of Transitional Boundary-Layer Flows," *AIAA Journal,* Vol. 11, April 1973, pp. 554-556.

[11] Patankar, S. V. and Spalding, D. B., *Heat and Mass Transfer in Boundary Layers,* CRC Press, Cleveland, Ohio, 1968.

[12] van Driest, E. R., "On Turbulent Flow Near a Wall," *Journal of Aeronautical Sciences,* Vol. 23, Nov. 1956, pp. 1007-1011, 1036.

[13] Shamroth, S. J. and McDonald, H., "Assessment of a Transitional Boundary Layer Theory at Low Hypersonic Mach Numbers," CR-2131, Nov. 1972, NASA.

[14] Warmbrod, J. D., Martindale, W. R., and Matthews, R. K., "Heat Transfer Rate Measurements on North American Rockwell Orbiter (161B) at Nominal Mach Number of 8," CR-120,029, Dec. 1971, NASA.

[15] Martindale, W. R., Matthews, R. K., and Trimmer, L. L., "Heat-Transfer and Flow Field Tests of the North American Rockwell/General Dynamics Convair Space Shuttle Configurations," AEDC-TR-72-169 (AD755354), Jan. 1973, Arnold Engineering Development Center, Arnold Air Force Station, Tenn.